THE QUASI-LINEAR RELAXATION OF THICK-TARGET ELECTRON BEAMS IN SOLAR FLARES

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ABSTRACT

The effects of quasi-linear interactions on thick-target electron beams in the solar corona are investigated. Coulomb collisions produce regions of positive gradient in electron distributions which are initially monotonic decreasing functions of energy. In the resulting two-stream instability, energy and momentum are transferred from electrons to Langmuir waves and the region of positive slope in the electron distribution is replaced by a plateau. In the corona, the timescale for this quasi-linear relaxation is very short compared to the collision time. It is therefore possible to model the effects of quasi-linear relaxation by replacing any region of positive slope in the distribution by a plateau at each time step, in such a way as to conserve particle number. The X-ray bremsstrahlung and collisional heating rate produced by a relaxed beam are evaluated.

Although the analysis is strictly steady state, it is relevant to the theoretical interpretation of hard X-ray bursts with durations of the order of a few seconds (i.e. the majority of such bursts).

1. Introduction

It is widely accepted that hard X-ray bursts observed during solar flares are produced by the bremsstrahlung of non-thermal electrons, but relatively few authors have considered the possible consequences of collective plasma effects on the dynamics of thick-target electron beams. Considerable attention has been paid recently to the importance of reverse current Ohmic losses due to collisional resistivity (e.g. Emslie 1980), but the effects of plasma wave generation resulting from beam instability (as described by quasi-linear theory) have been neglected by most authors. Emslie and Smith (1984) pointed out that the effect of Coulomb collisions is to produce regions of positive gradient in electron distributions which are initially monotonic decreasing functions of energy: this gives rise to the well-known "bump on

tail" instability, and Langmuir wave generation is set up.

In this paper we examine the effect of quasi-linear relaxation on the bremsstrahlung emission and collisional heating rate associated with an electron beam in the corona. In Section 2, the quasi-linear equations and their asymptotic solutions are discussed. In Section 3, the collisional energy loss rate of an electron in a warm target is used to infer the evolution of the electron beam in a collisionally dominated thick target. Numerical computations of the distribution function, with and without quasi-linear relaxation, are presented. Computations of the corresponding hard X-ray spectra and heating rates are presented in Section 4. In Section 5 we compare our results with those of previous authors and consider their implications.

2. The Quasi-linear Equations

In the following we will assume that the source region is a homogeneous fully ionized hydrogen plasma. The beam electrons will be assumed to be non-relativistic and to be streaming in one direction only: for simplicity, pitch angle scattering will be neglected. In order to simplify the quasi-linear equations, only Langmuir waves propagating in the streaming direction will be considered.

Let $f(v)$ and $W(v)$ denote respectively the electron velocity distribution (differential in velocity space) and the energy density in Langmuir waves (differential in phase velocity space). Then the quasi-linear equations may be written as (Melrose 1980)

$$\frac{df}{dt} = \frac{\pi \omega_p}{mn} \frac{\partial}{\partial v} \left[v \, W(v) \frac{\partial f}{\partial v} \right] \tag{1}$$

$$\frac{dW}{dt} = \frac{\pi \omega_p}{n} v^2 W(v) \frac{\partial f}{\partial v} \tag{2}$$

where m is the electronic mass, n is the ambient density and ω_p is the electron plasma frequency. In general, d/dt denotes the total (i.e. advective) time derivative. We will now argue that, if there exists a region of positive slope in the electron distribution (i.e. positive $\frac{\partial f}{\partial v}$, corresponding to wave growth) then quasi-linear interactions will dominate over Coulomb interactions in the corona. The wave growth rate associated with equation (2) is

$$\gamma_w = \pi \frac{\omega_p}{n} v^2 \frac{\partial f}{\partial v} \tag{3}$$

Now consider the situation shown schematically in Figure 1 in which a beam distribution is superimposed on a background Maxwellian, a region of substantial positive slope lying between v_1 and v_2. Putting $\frac{\partial f}{\partial v} \simeq \frac{f}{\Delta v}$ ($\Delta v = v_2 - v_1$) and defining

$$n_1 \equiv \int_{v_1}^{v_2} f \, dv \simeq f \, \Delta v$$
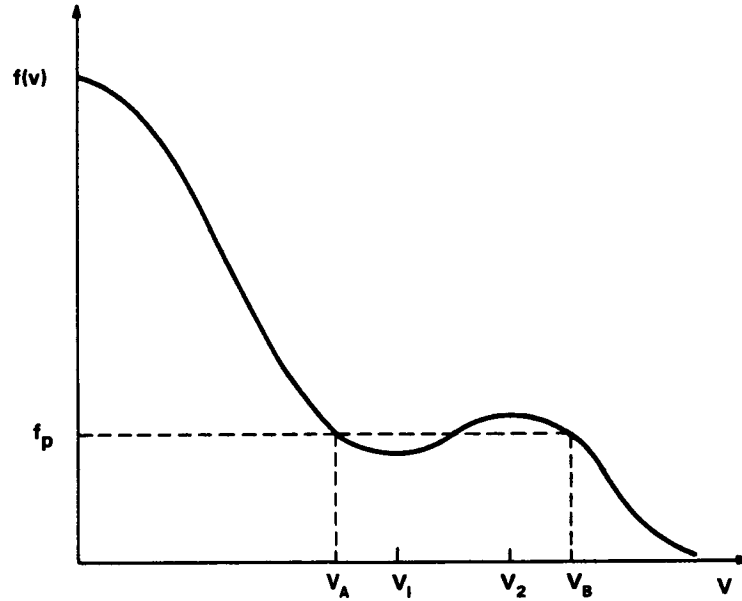
374

Figure 1. The form of the combined electron distribution giving rise to the "bump on tail" instability. A region of positive slope lies between velocities v_1 and v_2. The plateau of the relaxed distribution is defined by the three parameters v_A, v_B and f_p.

the growth rate may be written as

$$\gamma_w = \pi \, \omega_p \frac{n_1}{n} \left(\frac{v}{\Delta v}\right)^2 \qquad (3a)$$

while the collisional damping rate is given by (Ginzburg 1961)

$$\gamma_c \simeq \frac{5.5n}{T^{3/2}} \ln \left(10^4 \, \frac{T^{2/3}}{n^{1/3}}\right) \simeq 70 \, nT^{-3/2} \qquad (4)$$

where T is the electron temperature, and the logarithmic factor has been set equal to a constant with $T \sim 10^7 K$ and $n \sim 10^{10} \, cm^{-3}$ (i.e. typical coronal values). From equations (3a) and (4) we obtain, assuming $\Delta v \simeq v$ (a reasonable assumption in practice),

$$\Gamma \equiv \frac{\gamma_w}{\gamma_c} \simeq 2500 \, n_1 \left(\frac{T}{n}\right)^{3/2}. \qquad (5)$$

The value of n_1 depends principally on the total injected electron flux. For fluxes of the order of $10^{19} cm^{-2} s^{-1}$ (fluxes as large as this are required by the thick target interpretation of some hard X-ray bursts) it turns out that $n_1 \gtrsim 10^6 \, cm^{-3}$. Putting $T \sim 10^7 K$ and $n \sim 10^{10} \, cm^{-3}$ as before indicates

375

that $\Gamma \gtrsim 10^5$, so that the timescale for quasi-linear relaxation is extremely short compared to the collision time. This justifies the omission of collisional terms in equation (1) and enables us to model the effects of relaxation by applying the asymptotic solution of equations (1) and (2) (in the chromosphere, $\Gamma \ll 1$, so that the effects of quasi-linear relaxation may be neglected in that region).

The quasi-linear equations can only be solved numerically. Grognard (1975) obtained a solution of the one-dimensional quasi-linear equations including spontaneous emission terms, with the initial conditions of zero wavelevel and a Gaussian electron distribution. As expected, the asymptotic solution for f(v) is a plateau in velocity space. However, Grognard points out that the time for the plateau to be formed is considerably longer than the characteristic growth time associated with equation (3). In fact the asymptotic solution is only valid for times $\tau \gtrsim 100/\gamma_w$: this does not, however, alter our conclusion that quasi-linear interactions dominate over Coulomb interactions in the Corona.

Although a numerical treatment of equations (1) and (2) is essential for studying the details of the relaxation process, the asymptotic value of the wavelevel may be readily determined for any given initial distribution f(v,0). Melrose (1980) obtained such an asymptotic solution in the case of a delta function velocity distribution. An explicit calculation of the wavelevel to be expected is important because of the (possibly observable) plasma radiation it excites. In fact Emslie and Smith (1984), on the basis of their calculation, estimated that the wavelevel would give rise to a microwave flux far in excess of that observed in a typical event, unless the microwaves are strongly gyroresonance absorbed. A convincing explanation of this anomaly, consistent with the thick-target model, does not yet exist.

3. The Evolution Of The Electron Distribution with Depth

We will assume that instability (i.e. wave generation) will always occur whenever a region of positive slope appears in the electron distribution. The combined distribution function is given by

$$f(v) = f_b(v) + f_o(v) \qquad (6)$$

where f_o is the distribution function for the background plasma and f_b is the distribution function of a vertically injected beam of electrons. Following Knight and Sturrock (1977) we will consider the beam distribution corresponding to the injected differential energy spectrum

$$F_o(E_o) = (\delta - 1) F_{oo} \frac{E_{oo}^{\delta-1}}{(E_{oo} + E_o)^{\delta}} \qquad (7)$$

where F_{oo} is the total injected flux (cm^{-2}), and E_{oo} and δ are constants. Neglecting pitch angle scattering, the instantaneous steady state electron energy spectrum F(E) is given by the continuity equation

$$F(E) dE = F_o(E_o) dE_o \qquad (8)$$

and the beam distribution function is given by

$$v f_b(v) \, dv \;=\; F(E) \, dE$$

i.e.
$$f_b(v) \;=\; m F(E). \tag{9}$$

To evaluate E_o from any given E we require the beam electron energy loss rate. It turns out that the relaxation process affects $f(v)$ at energies of typically a few kT (depending on the model parameters, such as F_{oo}), and therefore it is not self-consistent here to use the cold target formula for the energy loss rate assumed by, for example, Brown (1971). The energy loss rate of an electron in a warm fully ionized target, taking into account only electron-electron collisions (cf Emslie 1978), is given by

$$\frac{dE}{dt} \;=\; -\,\frac{2\pi e^4 \, \ell n \Lambda}{E}\, nv\, (\Phi(x) - 2x\Phi'(x)) \tag{10}$$

(Spitzer 1962) where $\ell n \Lambda \simeq 25$ is the Coulomb logarithm, e is the electronic charge, $x = (E/kT)^{1/2}$ and Φ is the error function. Writing $\psi(x) = \Phi(x) - 2x\Phi'(x)$, $K = 2\pi e^4 \ell n \Lambda$ and defining the usual column depth variable

$$N \;=\; \int_0^z n \, dz' \;=\; nz$$

equation (10) becomes

$$\frac{dE}{dN} \;=\; -\frac{K}{E}\psi(x). \tag{10a}$$

The numerical solution of equation (10a) yields E_o for prescribed E, N. dE_o/dE is then given by

$$\frac{dE_o}{dE} \;=\; \frac{E}{E_o}\,\frac{\psi(x_o)}{\psi(x)} \tag{11}$$

where $x_o = (E_o/kT)^{1/2}$.

Using equations (9), (10a), and (11) we can evaluate $f(v)$ (neglecting the effects of quasi-linear relaxation) for any prescribed set of parameters (F_{oo}, E_{oo}, δ, T and n). Quasi-linear relaxation can be incorporated in the scheme in the following way: if a region of positive slope is found in the combined distribution, it is immediately replaced by a plateau which conserves particle number. The three parameters which define the plateau are, as indicated in Figure 1, v_A, v_B and f_p. These are (uniquely) defined by the condition that

$$\int_{v_A}^{v_B} (f(v) - f_p)\, dv \;=\; 0. \tag{12}$$

Although there are three unknown parameters, only one of these is independent: they may all be readily determined numerically. The smoothed-out distribution function minus the background Maxwellian can then be taken to be the new $F_o(E_o)$, and the distribution function $F(E)$ corresponding to the subsequent N-step can be evaluated as before. $E_o - E$ is thus the energy lost by an

electron in a single N-step. If E_o lies in the plateau region then

$$m\, F_o(E_o) = f_p - f_o(E_o). \qquad (13)$$

Otherwise, $F_o(E_o)$ is given by the function $F(E)$ as evaluated in the previous step.

In Figure 2 we present numerical computations of the combined distribution function $f(v)$, for typical thick-target parameters, at a depth of 10^{21} cm^{-2}. The plateau formed by relaxation extends from 13 keV to 130 keV.
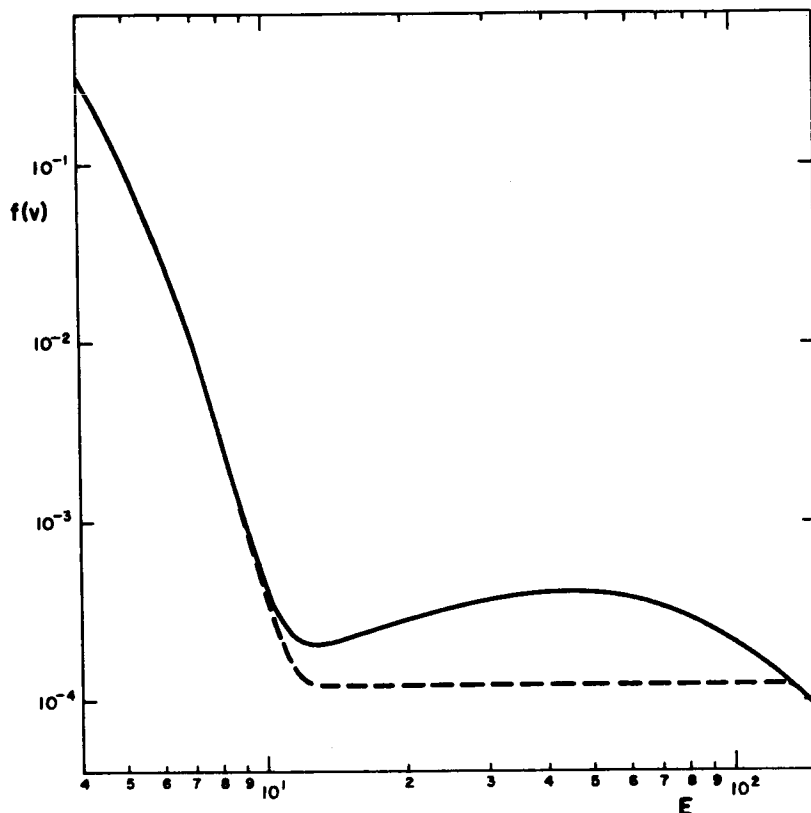

Figure 2. The combined distribution function for the model parameters $T = 10^7$K, $n = 10^{11}$ cm^{-3}, $F_{oo} = 10^{19}$ cm^{-2} s^{-1}, $E_{oo} = 20$ keV, $\delta = 4$. The column depth is 10^{21} cm^{-2}. The dotted line shows the plateau formed by relaxation. (f is measured in electrons cm^{-3} (cms^{-1})$^{-1}$ and E is measured in keV.)

4. The Bremsstrahlung Emission and Heating Rate

Figure 3 shows the local bremsstrahlung spectrum corresponding to the distribution function shown in Figure 2 (the non-relativistic Bethe-Heitler cross-section, averaged over solid angle, was used). The dotted line shows the spectrum obtained by including quasi-linear relaxation. It may be seen
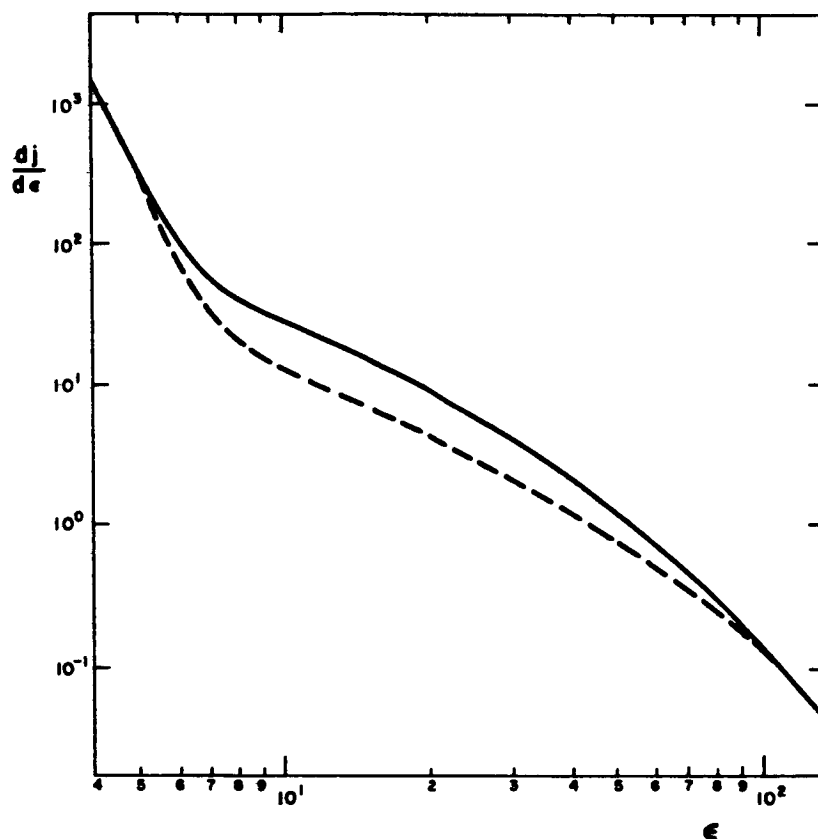
Figure 3. X-ray bremsstrahlung emissivity spectrum corresponding to the distribution function shown in Figure 2. (dj/dε is measured in photons cm^{-3} s^{-1} keV and ε is measured in keV.)

that relaxation has relatively little effect on the X-ray spectrum. Qualitatively, the emissivity is reduced in an energy range corresponding roughly to the plateau region in the electron distribution: the reduction is never more than about 50%. If the X-ray emissivity is integrated over the source volume, the overall effect of relaxation on the spectrum is much smaller: the reduction is ≲ 10%.

The above results are in qualitative agreement with those of Hoyng, Melrose and Adams (1979). They may be attributed to the "filtering" property of the Bethe-Heitler cross-section. What this means is that the source function f(v) is very sensitive to small perturbations on the photon spectrum. Conversely, different electron distributions can give rise to bremsstrahlung spectra which are almost identical (cf Brown 1975, Craig 1979).

In Figure 4 the collisional heating rate is shown as a function of column depth with the same beam and source parameters as before. The dotted line again indicates the case in which quasi-linear interactions are included. There is a considerable reduction in the heating rate for $N \gtrsim 10^{20}$ cm^{-2}

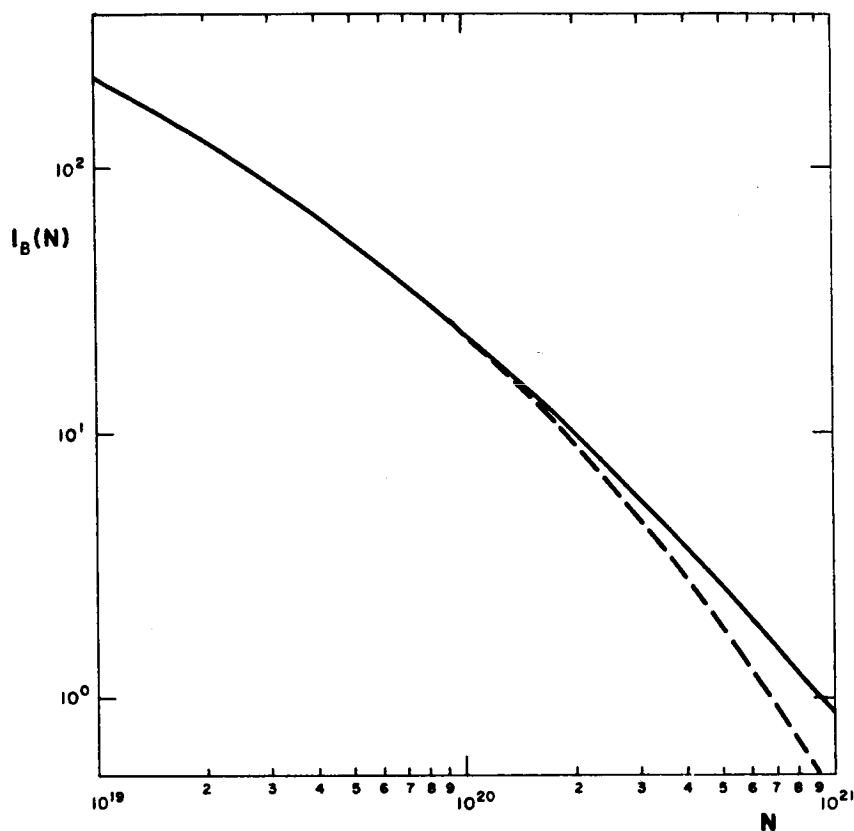Figure 4. Collisional heating rate as a function of column depth with the same beam and plasma parameters as before. (I_B is measured in ergs cm^{-3} s^{-1} and N is measured in cm^{-2}.)

($\sim 50\%$ at $N = 10^{21}$ cm^{-2}): this is to be expected since energy is being lost from the beam in the form of Langmuir waves. These waves are then damped and thereby heat the plasma: the total energy deposition rate is therefore greater than that indicated by the dotted line in Figure 4. The bremsstrahlung efficiency is consequently reduced and greater fluxes of electrons are required to explain hard X-ray bursts on the basis of a thick-target interpretation.

5. Discussion

As indicated previously, our results are consistent with those of Hoyng, Melrose and Adams (1979). These authors used a rather different technique, involving a Legendre series expansion of the three-dimensional quasi-linear equations. The form of the initial particle distribution was similar to that considered in this paper. It was found that bremsstrahlung spectra were not greatly affected by quasi-linear relaxation. It is quite likely, however,

that the total energy requirement of the thick-target model (with the inclusion of quasi-linear effects) may depend critically on the form of the injected electron spectrum. Our choice of a modified power law was governed by the aim of reproducing power law photon spectra, while at the same time having an acceptably small beam density to plasma density ratio. As mentioned previously, Melrose (1980) evaluated the asymptotic wave energy density in the case of a delta function injected particle distribution and showed that the particles eventually lose two thirds of their initial energy to waves. We would therefore expect the effects of quasi-linear relaxation on the energy requirement and the beam lifetime to be quite substantial in this case. For the beam and plasma parameters assumed in this paper, however, it appears that wave-particle interactions have an observationally negligible effect on the integrated bremsstrahlung emission.

There remains the problem of determining the wavelevel generated by a thick-target electron beam - this requires the numerical solution of the quasi-linear equations with collisional damping terms. The wavelevel so obtained may exceed the threshold for strong turbulence, with important consequences for the stability of both the beam and the reverse current (Vlahos and Rowland 1984, Rowland and Vlahos 1985).

Acknowledgement

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